EULER'S METHOD ON TI NSPIRE CAS

euler() program can be found in the Catalog:

From Guide for TI Nspire CAS:

euler() — Catalog >

euler(*Expr*, *Var*, *depVar*, {*Var0* *VarMax*}, *depVar0*, *VarStep* [, *eulerStep*]**)** ⇒ *matrix*

euler(*SystemOfExpr*, *Var*, *ListOfDepVars*, {*Var0*, *VarMax*}, *ListOfDepVars0*, *VarStep* [, *eulerStep*]**)** ⇒ *matrix*

euler(*ListOfExpr*, *Var*, *ListOfDepVars*, {*Var0*, *VarMax*}, *ListOfDepVars0*, *VarStep* [, *eulerStep*]**)** ⇒ *matrix*

Uses the Euler method to solve the system

$$\frac{d\,depVar}{d\,Var} = Expr(Var, depVar)$$

with $depVar(Var0)=depVar0$ on the interval $[Var0, VarMax]$. Returns a matrix whose first row defines the *Var* output values and whose second row defines the value of the first solution component at the corresponding *Var* values, and so on.

Expr is the right-hand side that defines the ordinary differential equation (ODE).

SystemOfExpr is the system of right-hand sides that define the system of ODEs (corresponds to order of dependent variables in *ListOfDepVars*).

ListOfExpr is a list of right-hand sides that define the system of ODEs (corresponds to the order of dependent variables in *ListOfDepVars*).

Var is the independent variable.

ListOfDepVars is a list of dependent variables.

{*Var0*, *VarMax*} is a two-element list that tells the function to integrate from *Var0* to *VarMax*.

ListOfDepVars0 is a list of initial values for dependent variables.

VarStep is a nonzero number such that **sign(***VarStep***)** = **sign(***VarMax*-*Var0***)** and solutions are returned at $Var0+i\cdot VarStep$ for all i=0,1,2,… such that $Var0+i\cdot VarStep$ is in $[var0, VarMax]$ (there may not be a solution value at *VarMax*).

eulerStep is a positive integer (defaults to 1) that defines the number of euler steps between output values. The actual step size used by the euler method is $VarStep / eulerStep$.

Differential equation:
y'=0.001*y*(100-y) and y(0)=10

$$\text{euler}(0.001\cdot y\cdot(100-y), t, y, \{0,100\}, 10, 1)$$

$$\begin{bmatrix} 0. & 1. & 2. & 3. & 4. & \ldots \\ 10. & 10.9 & 11.8712 & 12.9174 & 14.042 & \ldots \end{bmatrix}$$

To see the entire result, press ▲ and then use ◀ and ▶ to move the cursor.

Compare above result with CAS exact solution obtained using deSolve() and seqGen():

$$\text{deSolve}(y'=0.001\cdot y\cdot(100-y) \text{ and } y(0)=10, t, y)$$

$$y=\frac{100.\cdot(1.10517)^t}{(1.10517)^t+9.}$$

$$\text{seqGen}\left(\frac{100.\cdot(1.10517)^t}{(1.10517)^t+9.}, t, y, \{0,100\}\right)$$

$$\{10., 10.9367, 11.9494, 13.0423, 14.2189 \ldots$$

System of equations:

$$\begin{cases} y1' = -y1+0.1\cdot y1\cdot y2 \\ y2 = 3\cdot y2 - y1\cdot y2 \end{cases}$$

with $y1(0)=2$ and $y2(0)=5$

$$\text{euler}\left(\begin{Bmatrix} -y1+0.1\cdot y1\cdot y2 \\ 3\cdot y2-y1\cdot y2 \end{Bmatrix}, t, \{y1, y2\}, \{0,5\}, \{2,5\}, 1\right)$$

$$\begin{bmatrix} 0. & 1. & 2. & 3. & 4. & 5. \\ 2. & 1. & 1. & 3. & 27. & 243. \\ 5. & 10. & 30. & 90. & 90. & -2070. \end{bmatrix}$$

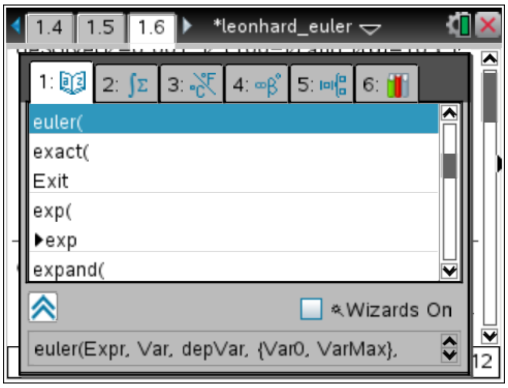
DETAILED INSTRUCTIONS TO USE EULER ON TI NSPIRE CAS

Example 1

Use Euler's method to find *y(4)* given that

$$\frac{dy}{dx} = x^2 - 2x \;,\; y(3) = 0.$$

From the Catalogue select Euler:



We need to enter the following:

y value from initial condition

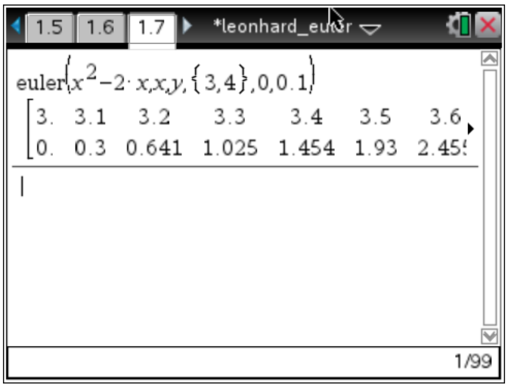
euler(x^2-2x,x,y,{3,4},0,0.1)

RHS of DE

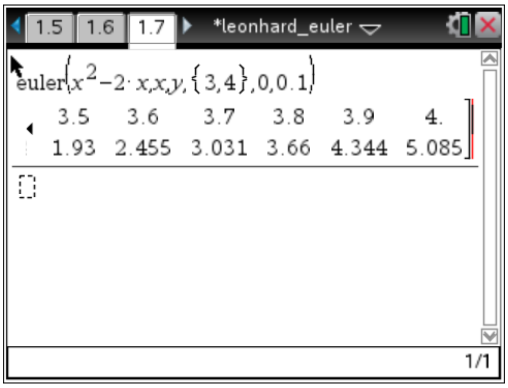
step

variables

initial *x*, final *x*

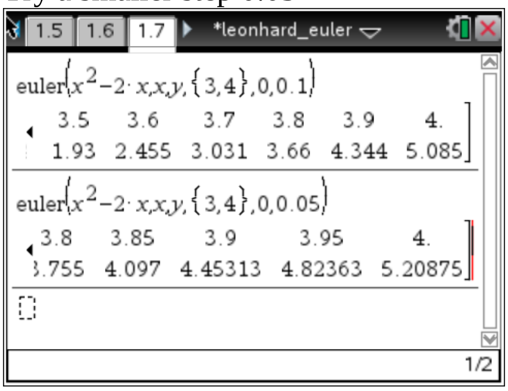


Use up and right arrows to see the right-hand end of the matrix box:



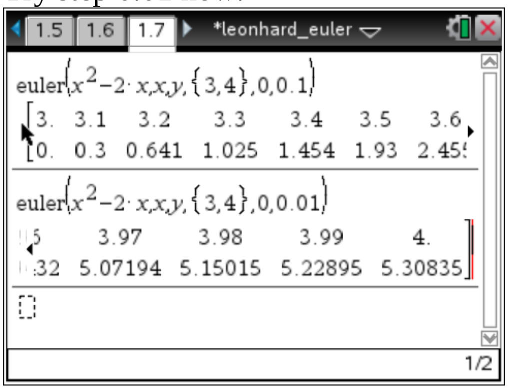
y(4)=5.085 with step 0.1

Try a smaller step 0.05



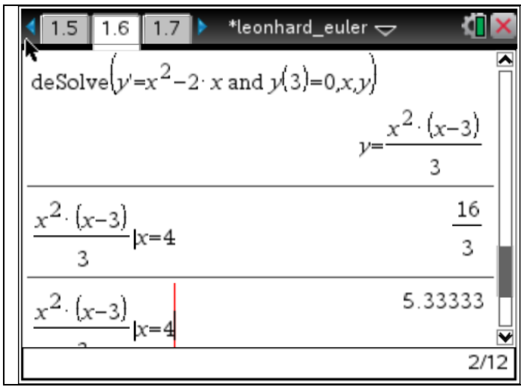
y(4)=5.20875 with step 0.05

Try step 0.01 now:



y(4)=5.30835 with step 0.01

Comparing the approximate solution with the actual solution:

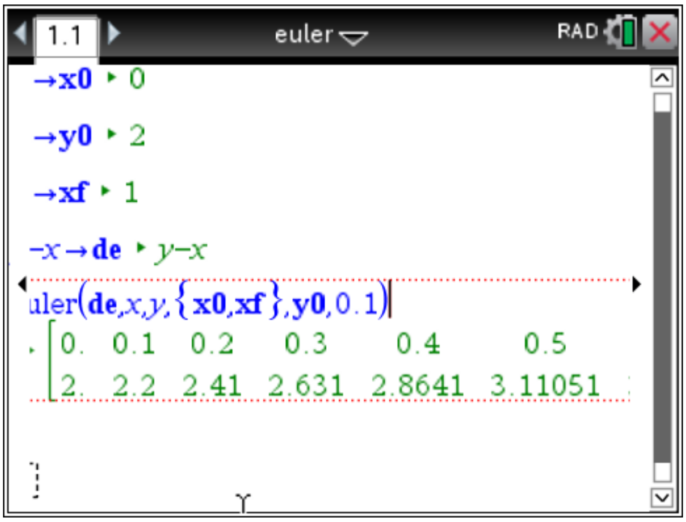


Clearly, the smaller the step, the more accurate the answer.

Note the Euler's method underestimates here.

Percentage error= - 0.47%

Setting in Notes page:



Example 2

a) Use Euler method to solve $\frac{dy}{dx} = 0.001y(100 - y),\ y(0) = 10$ to approximate the value of y at $x = 1$. Use the step 0.1 and 0.01

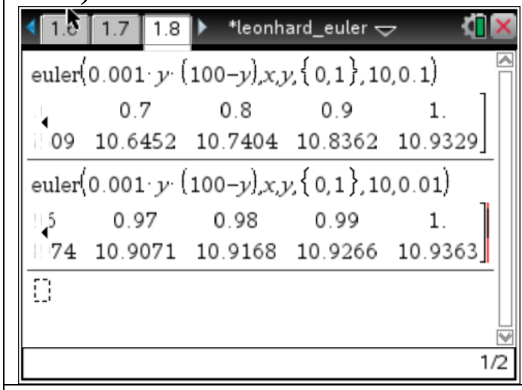
b) Use your CAS calculator to solve the DE and hence find *y(1).*

c) Comment on the accuracy of the solution.

d) Now repeat the above to find *y(2).*

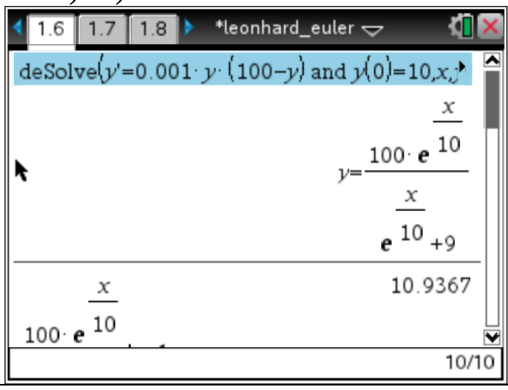
Solution:

a)



y(1)=10.9363 with step 0.01

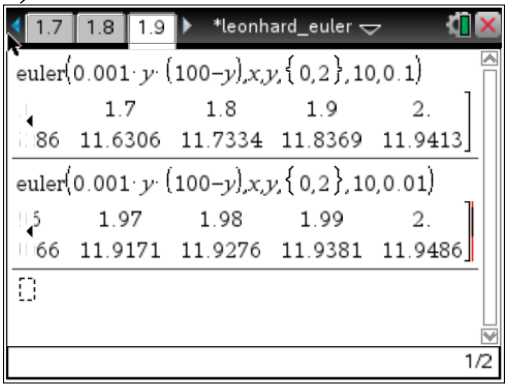
b) c)

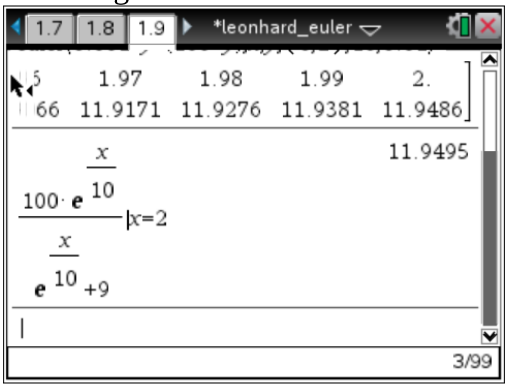


The actual value y(1)=10.9367

Percentage error = -.004%

d)





Percentage error = - 0.0075

Error increases when we approximate further from the initial point.

Video how to use Euler's method on TI Nspire:

http://www.youtube.com/watch?v=gvmYniqmSo4

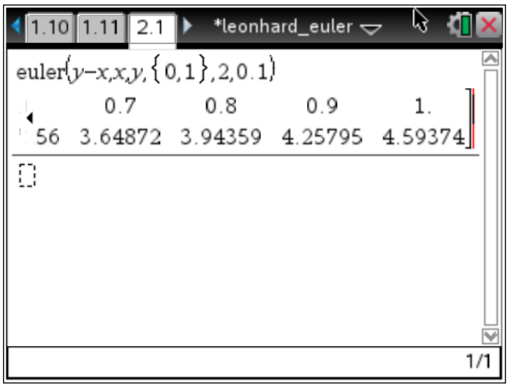
http://www.youtube.com/watch?v=MYI4YzuCj_w

Example 3: Use Euler's Method of Numerical Integration with a step size of 0.1 to find $y(1)$ if $y' = y - x$ and $y(0) = 2$.

The following spread sheet can be designed:

n	x_n	y_n	$f(x,y)h$	$y_{n+1} = y_n + f(x, y)\ h$
0	0	2	0.2	2.2
1	0.1	2.2	0.21	2.41
2	0.2	2.41	0.221	2.631
3	0.3	2.631	0.2331	2.864
4	0.4	2.864	0.2464	3.111
5	0.5	3.111	0.2611	3.372
6	0.6	3.372	0.2772	3.649
7	0.7	3.649	0.2949	3.944
8	0.8	3.944	0.3144	4.258
9	0.8	4.258	0.3358	4.59374
10	1.0	4.59374		

Using euler() program on TI Nspire CAS:

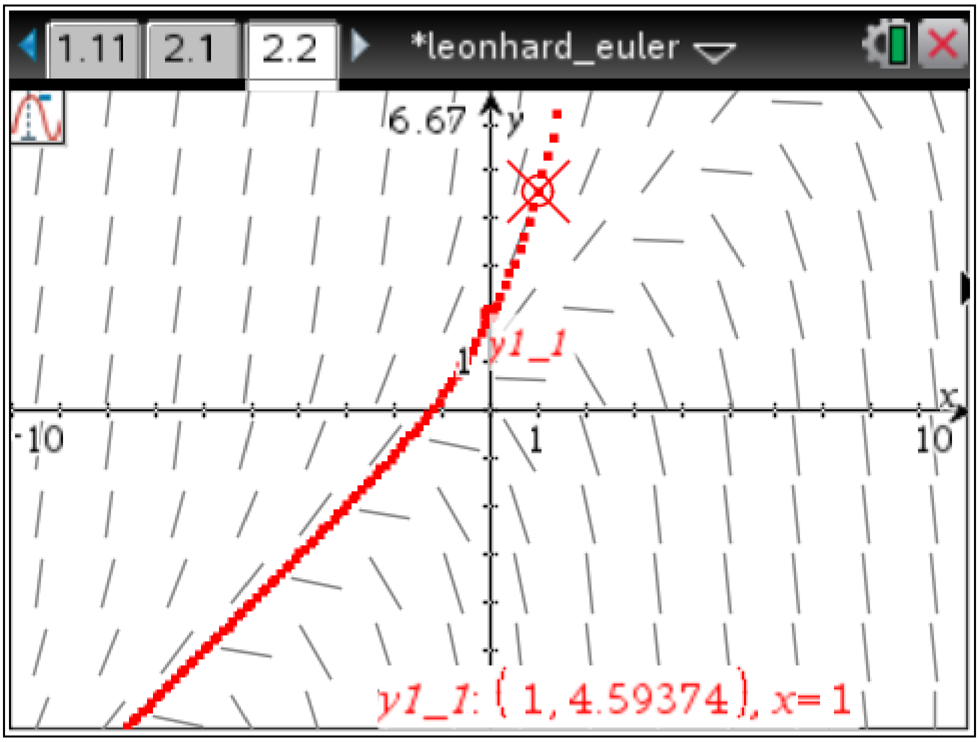


Use deSolve and find the percentage error. (Answer: about 2.64%)

Repeat with the step 0.05 (% error about 1.38%.)

Reducing the step size by half reduces the absolute error by roughly a half. However, reducing the step size also increases the amount of computation thereby increasing the potential for round-off error.

Solving graphically:
Plot a DE with initial conditions, check that Euler is selected, then Trace graph



<u>**VCAA Exam One 2007**</u>

Example 4

a. Use Euler's method to find y_2 if $\dfrac{dy}{dx} = \dfrac{1}{x}$, given that $y_0 = y(1) = 1$ and $h = 0.1$.
Express your answer as a fraction.

b. Solve the differential equation given in **part a.** to find the value y which is estimated by y_2. Express your answer in the form $\log_e(a) + b$, where a and b are positive constants.

(2 + 2 = 4 marks)