

Name: _____

IB MATHEMATICS HL Exponential and Logarithmic Functions Test

Paper 2 Technology Active

Time: 50 minutes

Marks: 40

Question 1 [1 + 1 = 2 marks]

Solve the following equations:

(a) $7^{5x-1} - 3 = 0$

(b) $\log_{10}(3x) + 2x = -4 - x$

Question 2 [2 + 1 + 2 + 3 = 9 marks]

Polonium-210 is a radioactive substance. The decay of polonium-210 is described by the formula $M = M_0 e^{-kt}$, where M is the mass in grams of polonium-210 left after t days, and M_0 and k are constants. At time $t = 0$, $M = 10g$ and at $t = 140$, $M = 5g$.

a. Find the values of M_0 and k .

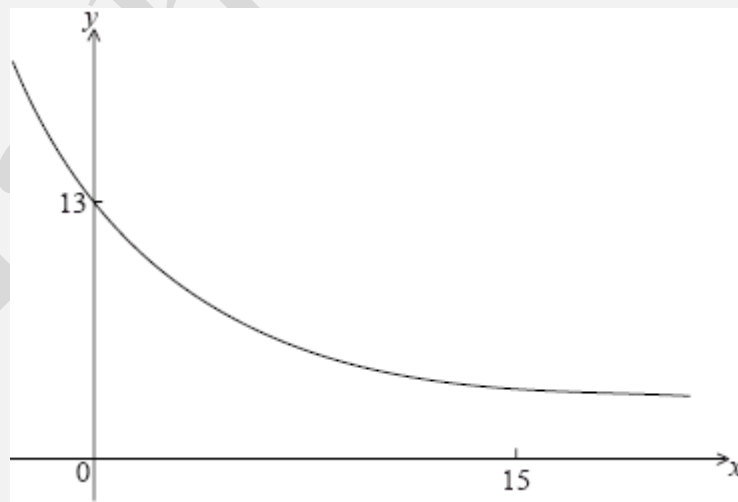
b. Find the mass of the polonium-210 after 70 days.

c. After how many days is the mass remaining 2g?

d. Sketch the graph of $M(t)$.

Question 3 [1 + 2 + 1 = 4 marks]

Let $f(x) = Ae^{kx} + 3$. Part of the graph of f is shown below.



The y-intercept is at (0, 13).

(a) Find the value of A .

(b) Given that $f(15) = 3.49$ (correct to 3 significant figures), find the value of k .

(c) Write down the equation of the horizontal asymptote of the graph f .

Question 4 [3 + 4 + 5 = 12 marks]

The functions f and g are defined by

$$f(x) = 1 + e^x, \text{ for } x \in \mathbb{R},$$
$$g(x) = \frac{1}{x}, \text{ for } x \in \mathbb{R} / \{0\},$$

a) i) Find the composite function $g \circ f$.

ii) Determine the range of the composite function $g \circ f$.

(b) Determine the inverse of the function $g \circ f$, clearly stating the domain.

(c) Sketch the graph of the inverse of the function $g \circ f$. Mark axes intercepts and equations of asymptotes.

Mathexams

Question 5 [3 + 3 + 4 = 10 marks]

A city is concerned about pollution, and decides to look at the number of people using taxis. At the end of the year 2000, there were 280 taxis in the city. After n years the number of taxis, T , in the city is given by

$$T = 280 \times 1.12^n.$$

- (a) (i) Find the number of taxis in the city at the end of 2005.

- (ii) Find the year in which the number of taxis is double the number of taxis there were at the end of 2000.

- (b) At the end of 2000 there were 25 600 people in the city who used taxis. After n years the number of people, P , in the city who used taxis is given by

$$P = \frac{2560000}{10 + 90e^{-0.1n}}.$$

- (i) Find the value of P at the end of 2005, giving your answer to the nearest whole number.

- (ii) After seven complete years, will the value of P be double its value at the end of 2000? Justify your answer.

PTO

(c) Let R be the ratio of the number of people using taxis in the city to the number of taxis. The city will reduce the number of taxis if $R < 70$.

(i) Find the value of R at the end of 2000.

(ii) After how many complete years will the city first reduce the number of taxis?

Question 6 [4 marks]

Find the set of values of x for which $|0.1x^2 - 2x + 3| < \log_{10} x$.

END OF PAPER 2