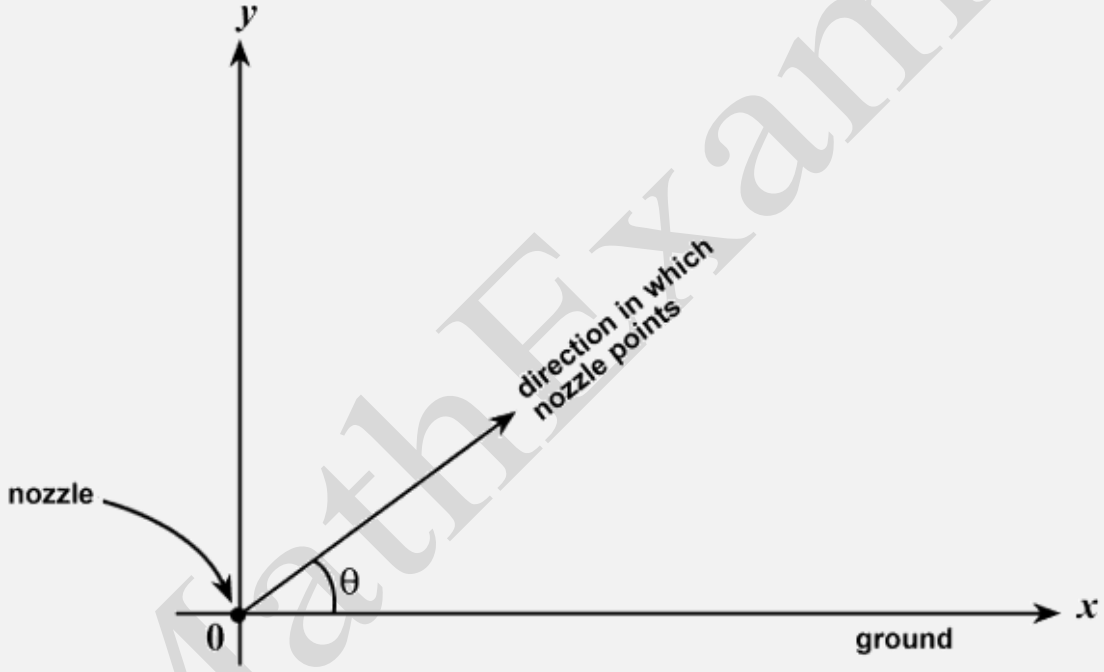
HOSE PROBLEM MODELLING ACTIVITY

You may remember what it was like when you and your friends played under a hose, squirting each other and moving the hose around watching the paths traced out by the water jets (and thus by each water droplet in each jet). Imagine yourself with a hose now (or go out and experiment with a hose). Suppose you turn the tap on so that the water is coming out of the nozzle at a speed of 10 m/s. Now suppose you are lying on the ground holding the hose with your arm outstretched in front of you so that the water jet is horizontal (along the ground). Let this direction be that of the positive x-axis of a Cartesian coordinate frame, with your hand near the origin, the nozzle of the hose being at the origin.

1. Imagine (or go out and observe) the shape of the water jets as you rotate your hand, by fixed amounts, until the jet is vertical. With reference to the Cartesian axes shown below, draw a sketch, based on your intuition or your observation, of the shape of a typical jet. What type of curve do you expect such a jet to trace out?



Suppose that the water jet leaves the nozzle at a given angle θ ($0° < \theta < 90°$) to the horizontal, and assume that air resistance against the jet is negligible. Then it can be shown that, with reference to the axes above, the curve traced out by the jet has the equation

$$y = ux - 0.049\,(1+u^2)x^2 \text{ where } u = \tan\theta.$$

Let us call this equation (i). The 0.049 is related to the initial speed of the water, 10 m/s, and acceleration due to gravity of 9.8 m/s^2.

2. Open the file Hose.tns to your TI Nspire CAS calculator. Go to page 1.2. Drag the marker on the slider to change the angle θ (called 'a'on a calculator). Comment on how the height and range change when the angle is varied from 0 to 89 degrees.
Hint: Make sure hat your calculator has Angle settings in degrees for this document (also Degrees for Graphs & Geometry).

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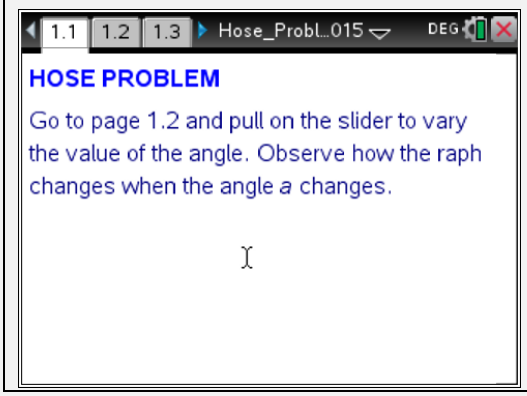
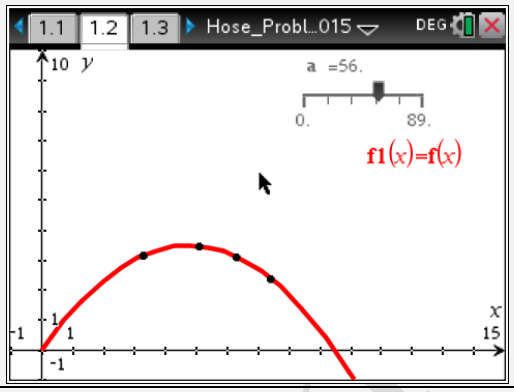
Define the function as follows (Menu 1: Action: Define): Assign the value of 30 degrees to pronumeral '*a*'.	Sketch the graph for a = 30 by inserting a Graph page. Go to Menu: 6: Analyze Graph 3: Maximum and mark the lower and upper bound.
	

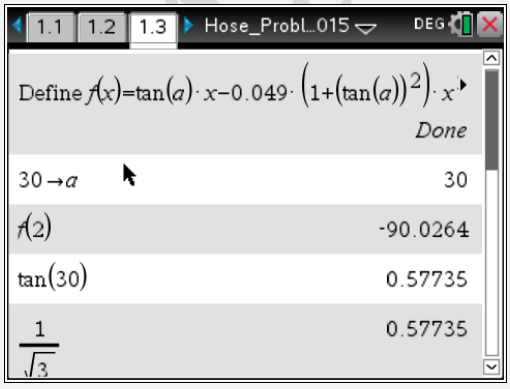
3. a. For the case where θ = 30°, find the position of the highest point reached by the jet and the position of the point where the jet strikes the ground, and sketch the graph of the curve traced out by the jet.

b. On the same set of axes, draw curves corresponding to three other angles θ, in each case locating carefully the highest point and the point where the jet strikes the ground.

4. a. Explain briefly why equation (i) cannot be applied directly to the case where θ = 90°; that is, where the nozzle is pointed vertically upwards.

b. By considering angles close to 90°, or otherwise, estimate the highest point reached by a jet which goes vertically upwards. Mark this point on the axes used in **2** above.

5. a. Imagine a high vertical wall at a horizontal distance of 1 metre from the nozzle. For the case where θ = 30° calculate the height at which the jet would strike the wall. Hint: Calculate the value of *f(1)* as shown below:



b. For each of the cases which you considered in **2.a.** calculate the height at which the jet would strike the wall described in **4.a.**

c. For one particular value of θ, the jet will hit this wall at a higher point than for any other value. Find this height.

d. Now imagine a high vertical wall at a horizontal distance of 5 metres from the nozzle. Find the highest point on this wall which could be reached by a jet.

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6. If the jet does **not** meet any obstacles on the way, then the furthest point on the horizontal ground that can be reached by a jet varies as the angle θ varies. Estimate the furthest point on the ground that a jet can reach.

7. Imagine all the jets simultaneously for all angles θ such that $0^\circ < \theta \leq 90^\circ$, like a fountain. The resulting sheet of water has a definite shape and boundary.

a. Sketch the boundary curve on the set of axes which you used in **2** above.

b. Discuss how you might go about finding the equation of the curve of the boundary shape.

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