ANSWERS to Interval of Convergence Exploration.

1. $\ln x = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \frac{1}{4}(x-1)^4 + \ldots$

2.

n	t_n	$\left\lvert \frac{t_{n+1}}{t_n} \right\rvert$
1	0.6	0.3
2	$-\frac{0.6^2}{2}$	0.4
3	$\frac{0.6^3}{3}$	0.45
4	$-\frac{0.6^4}{4}$	0.48
5	$\frac{0.6^5}{5}$	0.5
6	$-\frac{0.6^6}{6}$	0.5142…
7	$\frac{0.6^7}{7}$	0.525
8	$-\frac{0.6^8}{8}$	0.5333…
9	$\frac{0.6^9}{9}$	0.54

3. Shown in the table above.
4. The ratio appears to be approaching 0.6.
5. $\left\lvert \frac{t_{n+1}}{t_n} \right\rvert = \frac{n}{n+1} \times 0.6$
6. $S_4(1.6) = 0.4596$
7.

$|t_5| = 0.015552$

$|t_6| = 0.007776$

$|t_7| = 0.003999\ldots$

$|t_8| = 0.00209952$

$|t_9| = 0.0011\ldots$

8. $|t_5| = 0.015552(1 + 0.7 + 0.7^2 + 0.7^3 + 0.7^4)$

9. Geometric series converges to $\frac{0.015552}{1-0.7} = 0.05184$

10. Each term $|t_n| = \frac{0.6^n}{n}$ in the tail for $n \geq 5$ is less than $\frac{0.6^n}{5}$, and hence less than $\frac{0.6^n}{5} \times 0.7^{n-5}$, which is in turn less than $|t_5| \times 0.7^{n-5}$. But each such term, $|t_n|$ is less than the corresponding term in the geometric series in question 8. Therefore the sum of the absolute values in the tail is less than the sum of the geometric series, 0.05184, so 0.05184 is an upper bound for the sum of the terms in the tail.

11. The terms in the tail of ln4 have absolute value $\frac{1}{n} \times 3^n$, but bounding each term above by $\frac{1}{5} \times 3.1^n$ gives a geometric series which does not converge.

12. $\left|\frac{t_{n+1}}{t_n}\right| = \frac{n}{n+1}|x-1|$

13. $L = |x-1|$

14. $L < 1 \Rightarrow 0 < x < 2$

15. Radius of convergence is 1.