Interval of Convergence and Radius of Convergence Exploration

Objective: Find an interval of values of *x* for which the Taylor series for $\ln x$ converges.

1. Write the first few terms of the Taylor series for $\ln x$ expanded about $x = 1$.
2. Following the pattern in the series above, write the first nine terms of the series for $\ln 1.6$.

n	t_n	
1		
2		
3		
4		
5		
6		
7		
8		
9		

3. Put another column in the table above that shows the absolute value of the ratio of successive terms, $\left|\frac{t_{n+1}}{t_n}\right|$.
4. Do the ratios of terms seem to be approaching a limit as *n* becomes large? If so, what does this limit seem to be? If not, tell why not.
5. If you do not simplify the values of t_n in question 2, a pattern shows up in the ratios that allows you to answer the question 4 very easily. Find this pattern.
6. Find the fourth partial sum of the series for $\ln 1.6$.
7. Write the absolute values of the first five terms of the tail of the series after S_4.
8. Write the first five terms of the geometric series with first term $|t_5|$ and common ratio 0.7.
9. To what number does the geometric series in question 8 converge?
10. Explain why the number in question 9 is an upper bound for the sum of the absolute values of the terms in the tail of the $\ln 1.6$ series.
11. Explain why the technique of this exercise could not be used to find the upper bound for the tail of the ln series if 4 were substituted for *x*.
12. Write the formula for t_n in the Taylor series for $\ln x$ expanded about $x = 1$. Then write a formula for the absolute value of the ratio of terms $\left|\frac{t_{n+1}}{t_n}\right|$.
13. Find the limit of the fraction in question 12 as n approaches infinity. You should realize that *x* is a constant with respect to n and can be treated as such when you take the limit.
14. If the limit, L, in question 13 is less than 1, you can always find a convergent geometric series with common ratio between L and 1 that forms an upper bound for the tail of the ln series. By appropriate algebra, find an interval of *x*-values for which the ln series will converge if *x* is in this interval.
15. The interval you found in question 14 is called the **interval of convergence**. The distance from the middle of this interval to one of its endpoints is called the **radius of convergence**. What is the radius of convergence for $\ln x$ expanded about $x = 1$?