SUMMARY OF CONVERGENCE TESTS

NAME OF TEST	STATEMENT	COMMENTS
Divergence Test	If $\lim_{k\to\infty} u_k \neq 0$, then the series $\sum_{k=1}^{\infty} u_k$ diverges.	If $\lim_{k\to\infty} u_k = 0$, then the series $\sum_{k=1}^{\infty} u_k$ may either converge or diverge.
Integral Test	Let $\sum u_k$ be a series with positive terms, and let *f(x)* be the function that results when *k* is replaced by *x* in the general term of the series. If *f* is decreasing and continuous on the interval $[a, \infty)$, then $\sum_{k=1}^{\infty} u_k$ *and* $\int_a^{\infty} f(x)dx$ both converge or both diverge.	The test only applies to series that have positive terms. Try this test when *f(x)* is easy to integrate.
Comparison Test	Let $\sum_{k=1}^{\infty} a_k$ *and* $\sum_{k=1}^{\infty} b_k$ be series with non-negative terms and suppose that $a_1 \leq b_1,\ a_2 \leq b_2,\ a_3 \leq b_3, ..., a_n \leq b_n$ for all integers $m \geq k$. Then: (a) If the 'bigger' series $\sum_{k=1}^{\infty} b_k$ converges, then the 'smaller' series $\sum_{k=1}^{\infty} a_k$ also converges. (b) If the 'smaller' series $\sum_{k=1}^{\infty} a_k$ diverges, then the 'bigger' series $\sum_{k=1}^{\infty} b_k$ also diverges.	This test only applies to series with non-negative terms. Try this test as a last resort; other tests are often easier to apply.

Ratio Test	Let $\sum_{k=1}^{\infty} u_k$ be a series with positive terms and suppose that $a = \lim_{k\to\infty} \frac{u_{k+1}}{u_k}$ (a) If a < 1, then the series converges (b) If a > 1 or $a = \infty$, then the series diverges (c) If a = 1 , the test is inconclusive.	Try this test when u_k involves *k*th powers and factorials.
Limit Comparison Test	Theorem: Let $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ be series with positive terms. a) If $\lim_{n\to\infty} \frac{a_n}{b_n} = c > 0$ then $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ both converge or both diverge. b) If $\lim_{n\to\infty} \frac{a_n}{b_n} = 0$ and $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges. c) If $\lim_{n\to\infty} \frac{a_n}{b_n} = \infty$ and $\sum_{n=1}^{\infty} b_n$ diverges, then $\sum_{n=1}^{\infty} a_n$ diverges.	This is easier to apply that the comparison test, but still requires some skill in choosing the series $\sum_{n=1}^{\infty} b_n$ for comparison.
Alternating Series Test	The series $\sum_{n=1}^{\infty} (-1)^{n+1} u_n = u_1 - u_2 + u_3 - u_4 + \ldots$ converges if all three of the following conditions are satisfied: 1. The u_n's are all positive 2. $u_n \ge u_{n+1}$ for all $n \ge N$, for some integer N (u_n is a decreasing sequence) 3. $\lim_{n\to\infty} u_n = 0$	This test applies only to alternating series.

Ratio Test for Absolute Convergence	Let $\sum_{1}^{\infty} u_k$ be a series with non-zero terms such that $a = \lim_{k \to \infty} \left\| \frac{u_{k+1}}{u_k} \right\|$ a) The series converges absolutely if a<1 b) The series diverges if a>1 or $a = \infty$ c) The test is inconclusive if a = 1	The series does not need to have positive terms and does not need to be alternating. E.g. The alternating harmonic series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ converges conditionally.

SUMMARY ON SERIES CONVERGENCE FOR SOME SERIES

Convergent Series	Divergent Series
Geometric series with $\|r\| < 1$	Geometric series with $\|r\| \geq 1$
Telescoping series like $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$	The harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$
The series $\sum_{n=1}^{\infty} \frac{1}{n!}$	Any series $\sum_{k=1}^{\infty} a_k$ for which $\lim_{n \to \infty} a_n$ does not exist or $\lim_{n \to \infty} a_n \neq 0$
The p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ with $p>1$	The p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ with $p \leq 1$

INFORMAL PRINCIPLE :

1. Constant terms in the denominator of u_k can usually be deleted without affecting the convergence or divergence of the series.
2. If a polynomial in k appears as a factor in the numerator or denominator of u_k, all but the leading term in the polynomial can usually be discarded without affecting the convergence or divergence of the series.