

Name: \_\_\_\_\_

**MATHEMATICAL METHODS UNIT 2**  
**Differential Calculus Test 2016**

**Paper 1 (Technology Free)**

**Time Allowed 35 minutes**

**Total = 32 marks**

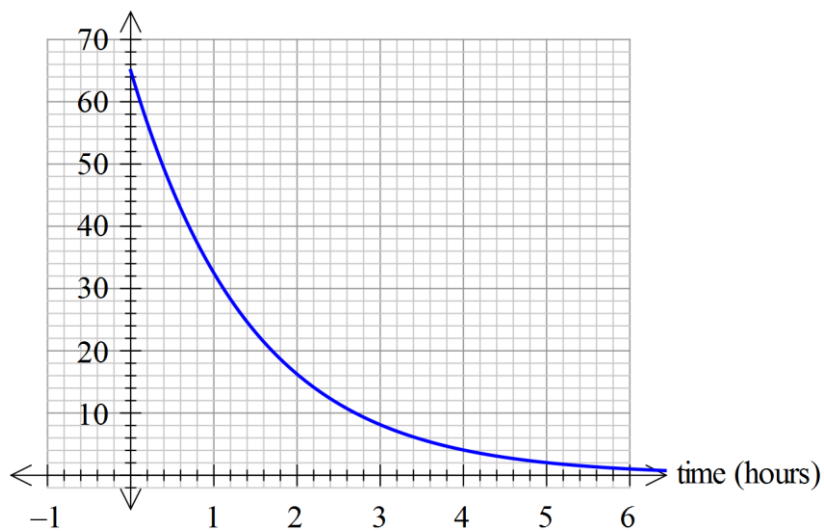
- A calculator or bound reference may *not* be used when completing this paper.
- Show all relevant working.

**Question 1**

After many surgical procedures, the drug pethidine is administered through an intravenous drip as a form of pain relief. The patient is initially given a high dose, which is then gradually decreased over a period of five days.

The graph below shows the concentration of pethidine administered in a particular case.

concentration (units/mL)



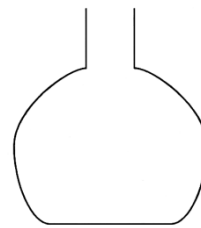
(a) Use the graph to estimate the average rate of change of concentration between days 1 and 3.

(b) Use the graph to estimate the instantaneous rate of change of concentration at time = 2 days.

2 + 2 = 4 marks

**Question 2**

Water is poured at a constant rate into the flask shown. Sketch a graph showing how the depth of water changes with respect to time.



2 marks

**Question 3**

Use  $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$  to find the derivative of  $f(x) = 3x^2 - 2x$  from first principles.

3 marks

**Question 4**

For each of the following find  $\frac{dy}{dx}$

a)  $y = 10x^3 - \frac{1}{2}x^6 + 5x$

b)  $y = -4x^{\frac{1}{3}}$

c)  $y = \frac{2}{x} - \sqrt{x}$

1 + 1 + 2 = 4 marks

**Question 5**

If the position of a particle moving in a straight line is given by the rule  $x(t) = -2t^2 + 8t + 3$ , where  $x$  is in centimetres and  $t$  is in seconds, find:

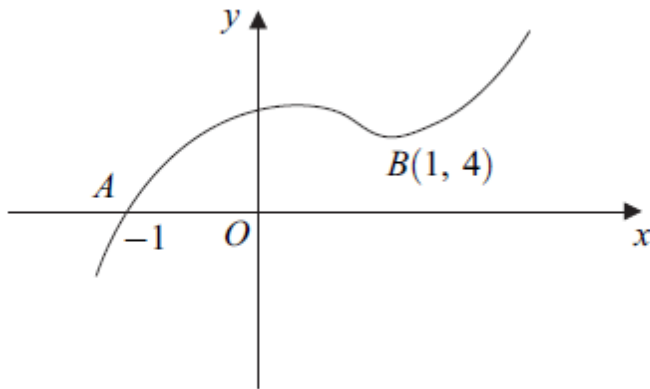
a) the velocity at any time  $t$  seconds.

b) where and when the velocity is zero.

1 + 2 = 3 marks

**Question 6**

The curve with equation  $y = x^5 - 3x^2 + x + 5$  is sketched below. The point  $O$  is the origin and the curve passes through the points  $A (-1, 0)$  and  $B (1, 4)$ .



- Find  $\frac{dy}{dx}$ .
- Find an equation of the tangent to the curve at the point  $A (-1, 0)$ .
- Verify that the point  $B$  is a stationary point on the curve.
- Complete the sign diagram below to show that there is a **minimum** at point  $B$ .

$x$	$\frac{1}{2}$	$1$	$2$
$f'(x)$			
$f(x)$			

1 + 2 + 1 + 2 = 6 marks

**Question 7**

Find the coordinates of the point on the parabola  $y = x^2 - 6x + 2$  at which the tangent is parallel to the line  $y = 2x - \frac{1}{3}$ .

4 marks

**Question 8**

The function  $f(x) = ax^3 - bx^2 + 27$  has a stationary point at  $(3, 0)$ .

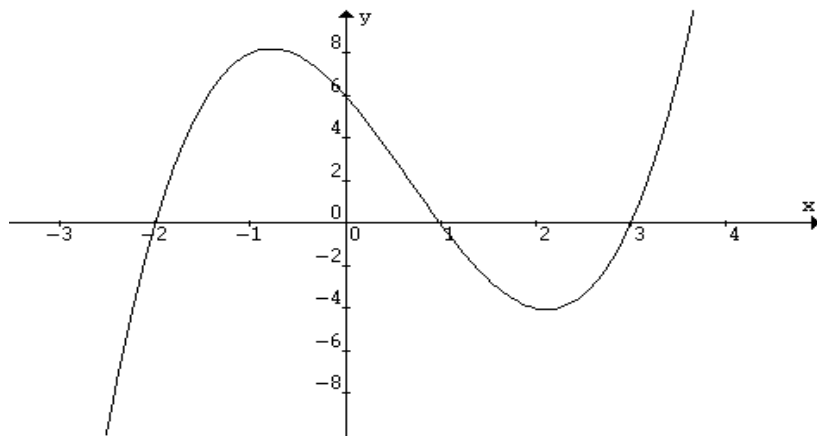
Find the values of  $a$  and  $b$ .

4 marks

**PTO**

### Question 9

For the graph shown, sketch the graph of the gradient function.



2 marks

END OF PAPER 1