

MATHEMATICAL METHODS UNIT 2
Differential Calculus Test 2016

Paper 2 (Technology Active)**Time Allowed 35 minutes****Total = 34 marks**

- A calculator and bound reference **may be used** when completing this paper.
- Show all relevant working.

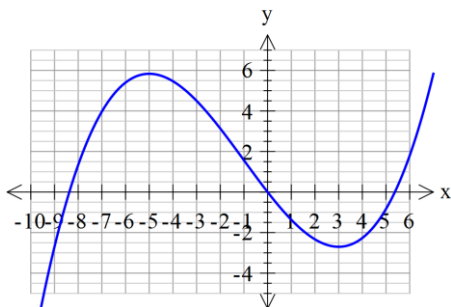
PART ONE: MULTIPLE CHOICE

Please circle the appropriate answer on the grid provided:

Question 1

$\lim_{x \rightarrow -2} \frac{4-x^2}{x+2}$ is equal to

- A. 4
- B. 0
- C. 2
- D. $\frac{1}{2}$
- E. 8

Question 2The graph of $f(x)$ is given below.The values of x for which $f'(x) > 0$ are:

- A $-5 < x < 3$
- B $x > 3$
- C $x < -5$
- D $x \leq -5$ and $x \geq 3$
- E $x < -5$ and $x > 3$

Question 3

The gradient of the function $y = f(x)$ at $x = 2$ is given by:

A. $\lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{2}$

B. $\lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h}$

C. $\frac{f(2+h) - f(2)}{2}$

D. $\frac{f(2+h) - f(2)}{h}$

E. $\lim_{x \rightarrow 0} \frac{f(2+h) - f(2)}{h}$

Question 4

If $f(x) = 3x^3 - 4x^2 - 2x$, then $f'(-2)$ is equal to:

A. 18

B. -4

C. 66

D. -36

E. 50

Question 5

The equation of the tangent to the curve $y = 4x - x^3$ at the point $(2,0)$ is:

A. $y + 8x = 16$

B. $y = 8x + 16$

C. $y = -8x + 2$

D. $y = 8x + 2$

E. $4y = -2x + 4$

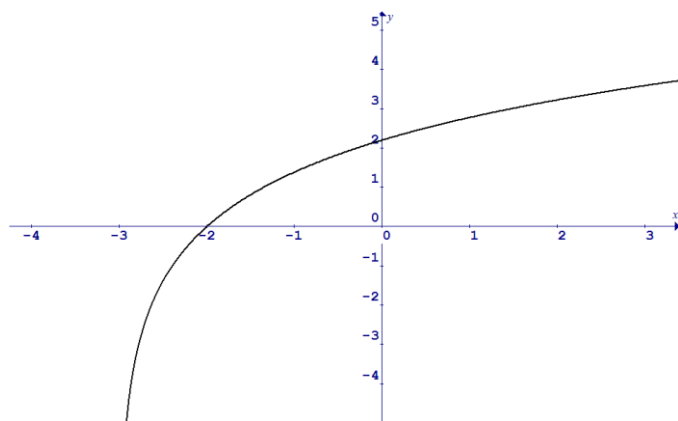
Question 6

Given that $f(x) = 2x^3 - 3x^2 - 12x$,
the value(s) of x for which $f'(x) = 0$ is/are

- A 0 and 3
- B 2
- C 2 and -1
- D -2 and 1
- E 0, 1 and 2

Question 7

The function graphed below has a positive gradient when



- A $x < -2$
- B $y > 2$
- C $x > -2$
- D always
- E never

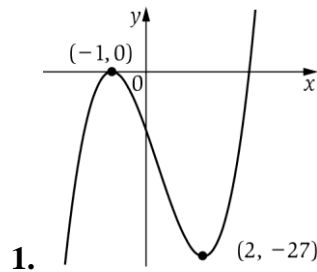
Question 8

The derivative of the function $f(x) = 4\sqrt{x} - \frac{4}{x}$ is:

- A. $\frac{4}{\sqrt{x}} - \frac{4}{x^2}$
- B. $\frac{4}{\sqrt{x}} - 4$
- C. $\frac{4}{\sqrt{x}} + \frac{4}{x^2}$
- D. $2\sqrt{x} + \frac{4}{x^2}$
- E. $\frac{2}{\sqrt{x}} + \frac{4}{x^2}$

Question 9

The graph of a cubic polynomial with rule $y = f(x)$ is shown below.



Which one of the following statements regarding the graph is **not** correct?

- A The graph has zero gradient at the points where $x = -1$ and $x = 2$.
- B The gradient of the graph is negative for $-1 < x < 2$.
- C The graph has two x -intercepts and two stationary points.
- D The graph has a local minimum at $(2, -27)$.
- E The graph has a stationary point of inflection at $(-1, 0)$.

Question 10

A colony of organisms multiplies at a rate according to $N(t) = 4t^2 + 1$. The average rate of increase between $t = 3$ and $t = 4$ is:

A 28

B 37

C 65

D 18

E 38

Section B: Extended Response Questions

Question 1

Find the equation of the tangent and normal to the curve $y = x^3 + 3x^2 - x + 1$ at the point $(1,4)$.

At what other point(s) does the tangent intersect the graph?

2 + 3 = 5 marks

Question 2

Water is being poured into a tank so that the volume, V mL, of water in the tank at time t minutes is given by

$$V(t) = \frac{1}{3} \left(8t^2 - \frac{t^3}{2} \right), 0 \leq t \leq 10.$$

a) Find the rate of flow of water into the tank at any time t .

b) Find the rate of flow of water into the tank at $t = 5$.

2 + 1 = 3 marks

Question 3

Let $s(t) = 5t^2 - 3t$ be the displacement function of a particle moving in a straight line, where t is in seconds and s is in metres.

a) Find the average velocity for the time interval $[0, 3]$.

b) Calculate the average speed in the first three seconds.

2 + 3 = 5 marks

Question 4 (11 marks)

A dog food manufacturer has to cut production costs. She wishes to use as little aluminium as possible in the construction of cylindrical cans. In the following diagram, h represents the height of the can in cm, and x represents the radius of the base of the can in cm.

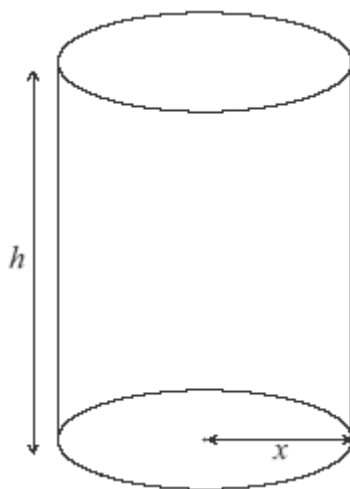


diagram not to scale

The volume of the dog food cans is 600 cm^3 .

(a) Show that $h = \frac{600}{\pi x^2}$.

2 marks

(b) (i) Find an expression for the curved surface area of the can, in terms of x . Simplify your answer.

(ii) Hence write down an expression for A , the total surface area of the can, in terms of x .

4 marks

(c) Differentiate A in terms of x .

3 marks

(d) Find the value of x that makes A a minimum.

1 mark

(e) Calculate the minimum total surface area of the dog food can.

1 mark

END OF PAPER 2