## Disappearing wombats

The mathematical techniques which might be required for this task include:

functions – exponential, logarithmic, domain and range, sketch graphs

calculus – differential equations, integration techniques, second derivatives

coordinate geometry – sketch graphs including asymptotic behaviour

**While any other prescribed methods are acceptable, the above techniques are considered particularly appropriate, and may feature in the test which will follow this task.**

Scientists are concerned that a species of wombat (*Wombaticus spuriosa*) may be in danger of extinction, as low numbers have been observed in the forests where they live. In an effort to protect the species from extinction, the Department of Conservation and Natural Resources decided to trap 200 wombats and move them to a remote island off the Australian coast. It is hoped that the wombat population will recover there safe from the problems on the mainland.

A simple mathematical model for the number $W$ of wombats on the island $t$ years after the initial 200 are settled there is given by

$$\frac{dW}{dt} = (m - n)W \qquad \text{(model 1)}$$

where $m$ is a positive constant related to the birth rate, and $n$ is a positive constant related to the death rate of the wombat population.

### Question 1

Suppose $m = 0.10$ and $n = 0.06$ in model 1. Find an expression for the number of wombats on the island after $t$ years. According to this model, what would the wombat population be

**i.** 10 years after the initial settlement?

**ii.** 100 years after the initial settlement?

### Question 2

Suppose $0.08 \le m \le 0.12$ and $0.06 \le n \le 0.09$ in model 1.

**a.** Find expressions for the number of wombats on the island after $t$ years for some carefully chosen combinations of possible values of $m$ and $n$ and sketch corresponding graphs showing how the wombat population changes over time.

**b.** According to this model, for what values of $m$ and $n$ will the wombat population

**i.** remain stable?

**ii.** increase without limit?

**iii.** eventually die out?

**c.** Comment on the limitations of model 1.

A more sophisticated mathematical model for the number $W$ of wombats on the island, $t$ years after the initial 200 are settled there, takes into account the availability of wombat food. Under this model, we have

$$\frac{dW}{dt} = (m - n - kW)W \qquad \text{(model 2)}$$

where $m$ and $n$ are again related to the birth rate and death rate respectively, and $k$ is a positive constant related to the amount of food available on the island.

**Question 3**

Explain the effect the term '–*k*W' has on the growth rate of the wombat population.

**Question 4**

Suppose $m = 0.10$, $n = 0.06$ and $k = 0.00005$ in model 2. Find an expression for the number of wombats on the island after $t$ years. According to this second model, what would the wombat population be 10 years and 100 years after the initial settlement? How does this compare with the first model? At what time $t$ is the wombat population increasing most rapidly?

Sketch a graph showing how the wombat population changes over time in this case. Comment on what your graph shows.

**Question 5**

Suppose $m = 0.10$, $n = 0.06$ and $0 < k \leq 0.001$ in model 2.

**a.** Find expressions for the number of wombats on the island after $t$ years for some carefully chosen values of $k$ and sketch corresponding graphs showing how the wombat population changes over time.

**b.** According to the second model, what will happen to the wombat population eventually, for each of your chosen values of $k$?

**c.** Generalise your answer to part **b.** for any positive constants $m$, $n$ and $k$, such that $m > n$.