Projectile Motion

Projectile Motion Subject to Gravity

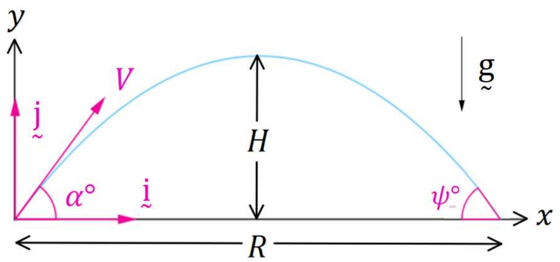
Given the initial speed V and the angled launched $\alpha°$ subject only to gravity and air resistance is negligible.

$$\underset{\sim}{a}(t) = \ddot{\underset{\sim}{r}}(t) = -g\underset{\sim}{j}$$

$$\underset{\sim}{v}(t) = \dot{\underset{\sim}{r}}(t) = -gt\underset{\sim}{j} + \underset{\sim}{v}(0)$$

$$= V\cos\alpha\,\underset{\sim}{i} + (V\sin\alpha - gt)\underset{\sim}{j}$$

$$\underset{\sim}{r}(t) = Vt\cos\alpha\,\underset{\sim}{i} + \left(Vt\sin\alpha - \frac{g}{2}t^2\right)\underset{\sim}{j} + \underset{\sim}{r}(0)$$



Cartesian Equation

$$y = x\tan\alpha - \frac{gx^2}{2V^2\cos^2\alpha} + r_y(0)$$

Features of Projectile Motion

Maximum Height H

The highest point the object reaches. At this point, the object has no vertical velocity.
Solving the $\underset{\sim}{j}$ component of the velocity vector equal to 0 will find the time.

Time of Flight T

Time taken from launch until the object lands. If the x-axis represents the ground, solving the $\underset{\sim}{j}$ component of the position vector equal to 0 will find the time of flight. If the ground is represented by a curve, find the intersection of the path of the object with curve representing the ground.

Range R

The distance from the where the object is launched to where it lands. If the x-axis represents the ground, the range is the $\underset{\sim}{i}$ component of the position vector at the when the object lands. If the ground is represented by a curve, find the magnitude of the position vector at the time when the object lands.
The maximum range occurs when the angle launched is $45°$.

Initial Velocity $\underset{\sim}{v}(0)$

The velocity vector at time equal to 0.

Initial Speed / Launch Speed $V = |\underset{\sim}{v}(0)|$

The magnitude of the velocity at the time equal to 0.

Landing Speed $|\underset{\sim}{v}(T)|$

The magnitude of the velocity at the time when the object lands.

Initial Angle / Launch Angle α

The direction of motion is parallel to the velocity. Therefore, the tangent of the initial angle is the ratio of the $\underset{\sim}{j}$ component of the velocity vector at the time when the object is launched and the $\underset{\sim}{i}$ component of the velocity vector at the time when the object is launched.
If given the Cartesian equation, the tangent of the initial angle is the gradient where $x = 0$.

$$\tan(\alpha) = \frac{\underset{\sim}{v}_y(0)}{\underset{\sim}{v}_x(0)} = \left.\frac{dy}{dx}\right|_{x=0}$$

Landing Angle

The direction of motion is parallel to the velocity. Therefore, the tangent of the landing angle is the ratio of the $\underset{\sim}{j}$ component of the velocity vector at the time when the object lands and the $\underset{\sim}{i}$ component of the velocity vector at the time when the object lands.

$$\tan(\psi) = \frac{\underset{\sim}{v}_y(T)}{\underset{\sim}{v}_x(T)}$$