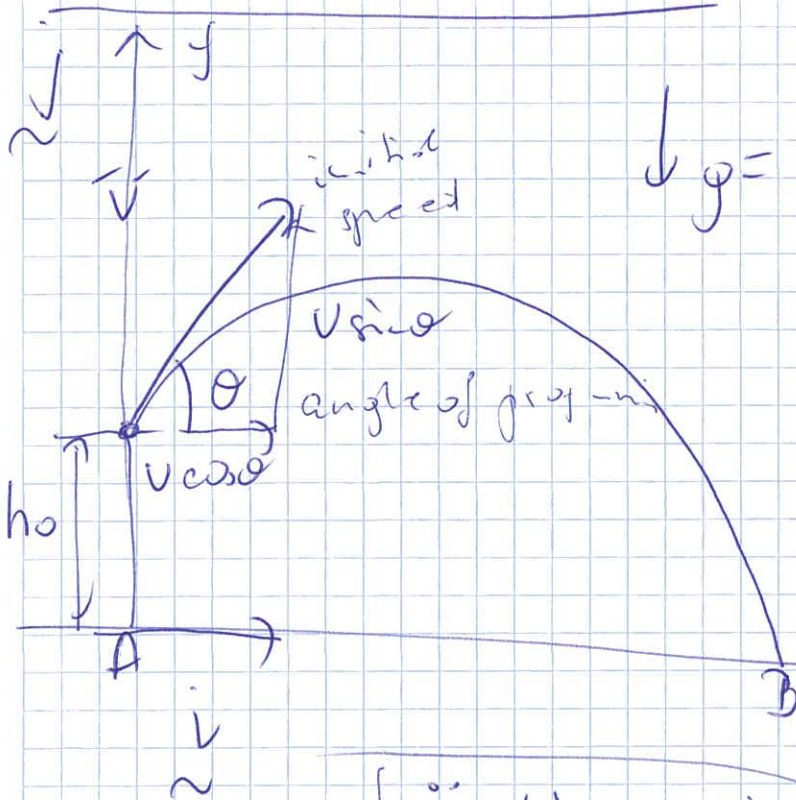
PROJECTILE MOTION

Air resistance is neglected.



$\downarrow g = 9.8\ ms^{-2}$

horizontal motion at constant speed $v\cos\theta$

vertical motion at constant acceleration $g = 9.8\ ms^{-2}$

$$\boxed{\ddot{\underset{\sim}{r}}(t) = -g\underset{\sim}{j}}$$ and $\dot{\underset{\sim}{r}}(0) = v\cos\theta\,\underset{\sim}{i} + v\sin\theta\,\underset{\sim}{j}$

$\underset{\sim}{r}(0) = h_0$

$$\Rightarrow \dot{\underset{\sim}{r}}(t) = -gt\underset{\sim}{j} + \underset{\sim}{c}$$

$$\Rightarrow v\cos\theta\,\underset{\sim}{i} + v\sin\theta\,\underset{\sim}{j} = -gt\underset{\sim}{j} + \underset{\sim}{c}$$

$$\boxed{\dot{\underset{\sim}{r}}(t) = v\cos\theta\,\underset{\sim}{i} + (v\sin\theta - gt)\underset{\sim}{j}}$$

$$\underset{\sim}{r}(t) = v\cos\theta\, t\,\underset{\sim}{i} + \left(v\sin\theta\, t - \frac{gt^2}{2}\right)\underset{\sim}{j} + \underset{\sim}{d}$$

$\underset{\sim}{d} = h_0$

$$\Rightarrow \underset{\sim}{r}(t) = v\cos\theta\, t\,\underset{\sim}{i} + \left(v\sin\theta\, t - \frac{gt^2}{2}\right)\underset{\sim}{j} + h_0\underset{\sim}{j}$$

$$\boxed{\underset{\sim}{r}(t) = v\cos\theta\, t\,\underset{\sim}{i} + \left(v\sin\theta\, t - \frac{gt^2}{2} + h_0\right)\underset{\sim}{j}}$$

MAXIMUM HEIGHT

from $\underset{\sim}{r}(t)$ make $\underset{\sim}{j}$ comp $= 0$

$$V\sin\theta - gt = 0 \quad \Rightarrow \quad t = \frac{V\sin\theta}{g}$$

now substitute in $\underset{\sim}{j}$ comp of $\underset{\sim}{r}(t)$ to find h_{max}.

RANGE

- max horizontal distance reached

(AB) make the y-comp of $\underset{\sim}{r}(t) = 0$ to find the time of flight (take the non-zero t) then sub t in x-comp of $\underset{\sim}{r}(t)$.

CARTESIAN EQUATION OF A PATH

$$\underset{\sim}{r}(t) = V\cos\theta\, t\,\underset{\sim}{i} + (V\sin\theta\, t - \tfrac{1}{2}gt^2 + h_0)\underset{\sim}{j}$$

$$x = V\cos\theta\, t \qquad y = V\sin\theta\, t - \tfrac{1}{2}gt^2 + h_0$$

$$t = \frac{x}{V\cos\theta}$$

$$y = V\sin\theta\,\frac{x}{V\cos\theta} - \frac{g}{2}\left(\frac{x}{V\cos\theta}\right)^2 + h_0$$

$$y = V\tan\theta\, x - \frac{g}{2}\frac{x^2}{V^2\cos^2\theta} + h_0$$

$$\boxed{y = x\tan\theta - \frac{1}{2}V\,\frac{gx^2\sec^2\theta}{2V^2} + h_0}$$