IB MATHEMATICS HL
Mathematical Investigation: The sum of cubes.

1. Find the sum of the series of the first n natural numbers

   $S_n = 1+2+3+4+....+n = \sum_{n=1}^{n} n$ (use arithmetic series) $\therefore \sum_{n=1}^{n} n =$

2. Let $\sum_{n=1}^{n} n^2$ denote the sum of squares of the first n natural numbers so that

   $\sum_{n=1}^{n} n^2 = 1^2 + 2^2 + 3^2 + ... + n^2$

3. Simplify the expression $x^3 - (x-1)^3 =$
4. Construct and complete the following table for $x = 1, 2, 3, ..., n$

x	x^3	$(x-1)^3$	$3x^2-3x+1$
1	1^3	0^3	$3(1)^2-3(1)+1$
2			
3			
4			
5			
$n-1$			
n			

5. Show that the expression $\sum x^3 - \sum (x-1)^3$ for the first five natural numbers is equal to 5^3 - 0^3 . (use the table above)
6. Hence, generalise the expression for $\sum n^3 - \sum (n-1)^3$ in terms of n (by subtracting appropriate columns in your table).
7. Using the table from Q4, show that $n^3 = 3\sum n^2 - 3\sum n + n$.
8. Therefore, find an expression for $\sum n^2$ in terms of n.
9. Let $\sum n^3 = 1^3 + 2^3 + 3^3 + ... + n^3$.
10. Simplify the expression $x^4 - (x-1)^4$.
11. Repeat the steps Q4 to Q8 and combine all previous findings to deduce an expression in terms of n for $\sum n^3$.
12. Prove the following conjecture $1^3 + 2^3 + 3^3 + ... + n^3 = \left[\frac{n(n+1}{2}\right]^2$ using mathematical induction.